

I U P U I
MATH CLUB TEASER #8

November 21, 2008
(due December 2, 2008)

SOLUTION

First let's settle on some notation to make the explanation easier to follow. Squares are divided into *corners*, *centers*, and *sides*. There are 4 corner squares, 4 center squares, and 8 side squares. Call a diagonal a *j*-diagonal if it consists of *j* squares; for instance, the 1-diagonals are exactly the corner squares. Also, number columns 1 to 4 from left to right, and number rows 1 to 4 from the bottom up

Note that every row, column, or diagonal flip changes an even number (0 or 2) of side squares. It follows that to obtain a board with only 0s, the initial board must have an even number of 0s in the side squares.

The following steps show that ANY initial board with an even number of 0s in the side squares can be flipped to a board with only 0s:

- Each 3-diagonal flips a unique center square; flip the necessary ones to set all center squares to 0.
- Using 2-diagonal flips, set the upper side squares to 0.
- Flip columns 1 and 4 if necessary to ensure that there is a 0 on the side squares of row 3.
- Using 2-diagonal flips, set the side squares of row 2 to 0.
- Only the side squares of row 1 can have 1s. If so, then both are 1, and a flip on row 1 sets them to 0.
- All center and side squares have 0 now. Flip individual corner squares as needed.

SOLVED BY:

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