

I U P U I
MATH CLUB TEASER #28

September 25, 2009
(due October 2, 2009)

SOLUTION

Danica can answer the exam in 4^{50} different ways. How many of these have exactly one wrong answer? There are 50 choices for the question with the wrong answer, and 3 wrong choices for that question, so there are 150 different ways of scoring a 49. In a similar way, the number of exams that have 0,1,2,3,4,5,6,7, and 8 wrong answers are:

1, 150, 11025, 529200, 18654300, 514858680,
11584320300, 218447182800, 3522460822650,

respectively. This means that the probability of passing the exam (answering at random) is

$$p = \frac{\text{Sum of the above}}{4^{50}} = \frac{1876513189553}{633825300114114700748351602688}.$$

If you throw a die looking for a 5, you may throw three times before getting 5 for the first time, or you may throw eleven times before getting your first 5. But in average, you expect to throw six times before getting 5. Similarly, the expected number of tries before Danica passes the exam is

$$\frac{1}{p} = \frac{633825300114114700748351602688}{1876513189553},$$

or about **337767570003116688** tries.

SOLVED BY:

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